VERIFICATION OF COMPUTER PROGRAM *LPILE* AS A VALID TOOL FOR DESIGN OF A SINGLE PILE UNDER LATERAL LOADING

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1. Introduction

Investigation of the response of a pile to lateral loading, sometimes called horizontal loading for vertical piles, was undertaken as early as deep foundations were employed. The early investigation produced a curve from a field experiment with a particular pile in a particular soil, showing lateral deflection of the pile head as a function of the applied load. Failure was usually taken as a deflection that was larger than could be tolerated in design and a depth to the *point of fixity* was computed. The pile in soil was replaced by a cantilever beam, fixed against rotation but free of soil, and the length of the beam, the depth to the point of fixity, was computed by using the lateral load at the pile head P_t, the deflection of the pile head y_t, and the bending stiffness of the pile E_pI_p. With the model, the engineer could easily compute the lateral load that would produce the desired deflection that the structure could tolerate.

Engineers realized that the point of fixity was only a computational tool that failed to represent the real behavior of a pile. Different depths to the point of fixity would be derived on the basis of the development of a plastic hinge, in one case, and a value of a limiting deflection, in another case. Further, the data were unavailable on which to base predictions for a variety of piles in a variety of soils.

A sustained effort to develop a rational method for the design of a laterally loaded pile began in the 1950's when energy companies started building offshore platforms to resist loads from hurricanes. The basic data came from tests of full-sized piles in various soils where the piles were instrumented over their full length for the measurement of bending moment. The *p-y* method of representing the response of the soil was developed

as a practical tool and could be implemented with the availability of the digital computer for solving the nonlinear differential equation. The method is described in detail in the following paragraphs.

2. Methods of Solution

Several methods have been published in technical literature for the analysis of piles loaded by lateral force. The linearly elastic solution presented by Poulos and Davis (1980) emphasizes the condition of continuity although the soil cannot be characterized as a linearly elastic material. The limit-equilibrium solution proposed by Broms (1965) can be applied to finding the ultimate lateral load at failure, but soil-structure interaction at lesser loads is not addressed.

The analysis of the laterally loaded pile by the finite-difference method has been developed extensively by a number of authors since 1960 (Reese and Matlock, 1960; Matlock and Reese, 1962; Matlock, 1963; Matlock and Ingram, 1963; Matlock and Haliburton, 1966; Reese, 1966; Reese, 1971; Matlock, 1970; Parker and Reese, 1971; Reese et al., 1974; Reese et al., 1975; Georgiadis, 1983). Their work proved the versatility and the theoretical applicability of the finite difference method in dealing with the highly nonlinear soil-pile-soil interaction.

The *p-y* method is being used extensively in the United States and elsewhere. To illustrate its use, references are cited from Italy (Jamilkowski, 1977), France (Baguelin et al., 1978), Britain (George and Wood, 1976), and Australia (Poulos and Davis, 1980).

The method is included in publications of the

- Federal Highway Administration, U.S. Department of Transportation (Reese, 1984), and adopted by most of the State Highway Departments in the United States;
- Det Norske Veritas in cooperation with the Wind Energy Department, Riso National Laboratory, "Guidelines for Design of Wind Turbines" (DNV, 2001);
- Det Norske Veritas, on Offshore Structures, (DNV, 1977); and the

- American Petroleum Institute (1993). The publications of the API have guided the design of onshore and offshore pile foundations in the United States and elsewhere.

3. Basic Equations for the *p-y* Method

The laterally loaded pile is modeled as shown in Fig. 1. The mechanisms shown to represent the soil depict the soil as a nonlinear material. The deformation of an elastic member under axial and lateral loading can be found by solving Eq. 1, the standard beam-column equation.

$$\frac{d^2}{dx^2}\left(E_p I_p \frac{d^2 y}{dx^2}\right) + P_x\left(\frac{d^2 y}{dx^2}\right) - p - W = 0 \quad \text{.....} (1)$$

where

P_x	=	axial load on the pile, F,
y	=	lateral deflection of the pile at point x along the length of the pile, L,
p	=	soil resistance per unit length, F/L,
W	=	distributed load along the length of the pile, F/L, and
$E_p I_p$	=	flexural stiffness, FL^2.

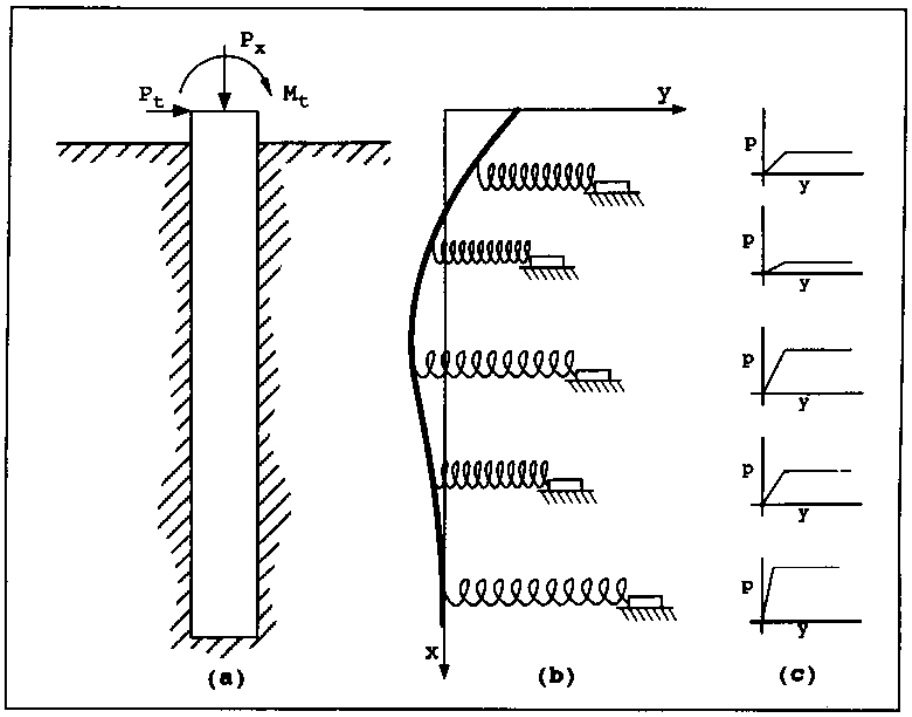

Fig. 1 Model of laterally loaded pile

A physical definition of the soil resistance p is given in Fig. 2. Figure 2a shows a profile of a pile that has been installed by driving or by some other method, and shows a thin slice of soil at some depth x_i below the ground surface. The assumption is made that the pile has been installed without bending so that the initial soil stresses at the depth x_i are uniformly distributed, as shown in Fig. 2b. If the pile is loaded laterally so that a pile deflection y_i occurs at the depth x_i the soil stresses will become unbalanced as shown in Fig. 2c. Integration of the soil stresses will yield the soil resistance p_i with units of F/L:

$$p_i = E_s y_i \qquad (2)$$

where

E_s = a parameter with the units F/L^2, relating pile deflection y and soil reaction p.

It is evident that the soil reaction p will reach a limiting value (and perhaps decrease) with increasing deflection. Furthermore, the soil strength in the general case will vary

with depth. Therefore, only in rare cases will E_S, sometimes called the soil modulus, is constant with depth.

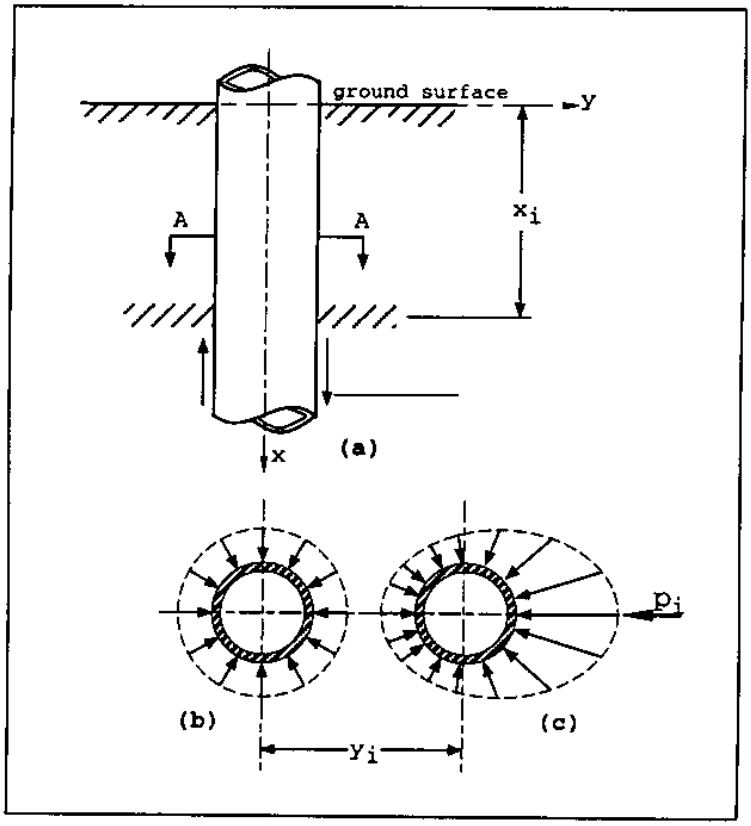

Fig. 2 Definition of p and y as related to the response of a pile to lateral loading

The bending stiffness E_pI_p of a metal pile will probably be constant for the range of loading of principal interest. However, the E_pI_p of a reinforced-concrete pile will change with the bending moment. In many designs, it is desirable to reduce the bending stiffness by reducing the wall thickness of a steel-pipe pile or by reducing the number of bars in a reinforced-concrete pile. Thus, in the general case the bending stiffness will not be constant with x nor with y.

In view of the nonlinearities of Eq. 1, numerical methods must be utilized to obtain a solution. The difference-equation method can be employed with good results. Eq. 3 is the differential equation in difference form, where the pile is subdivided as shown in Fig. 3.

$$y_{m-2}R_{m-1} + y_{m-1}(-2R_{m-1} - 2R_m + Qh^2) + y_m(R_{m-1} + 4R_m + R_{m+1} - 2Qh^2 + k_m h^4) +$$

$$y_{m+1}(-2R_m - 2R_{m+1} + Qh^2) + y_{m+2}R_{m+1} - W_m h^4 = 0 \quad (3)$$

where

$$R_m = E_m I_m, \quad (4)$$

$$k_m = E_{sm} \quad (5)$$

The pile is subdivided into n increments and $n+1$ equations can be written of the form of Eq. 3, yielding $n+5$ unknown deflections. Two boundary conditions at the bottom of the pile and two at the top of the pile allow for a solution of the $n+5$ equations with selected values of R and k. The value of n and the number of significant figures in y are selected to yield results with appropriate accuracy. The solution of the equations proceeds readily by Gaussian elimination. The value of n ranges from perhaps 50 to 200; on most computers double-precision arithmetic is necessary with about 15 significant figures.

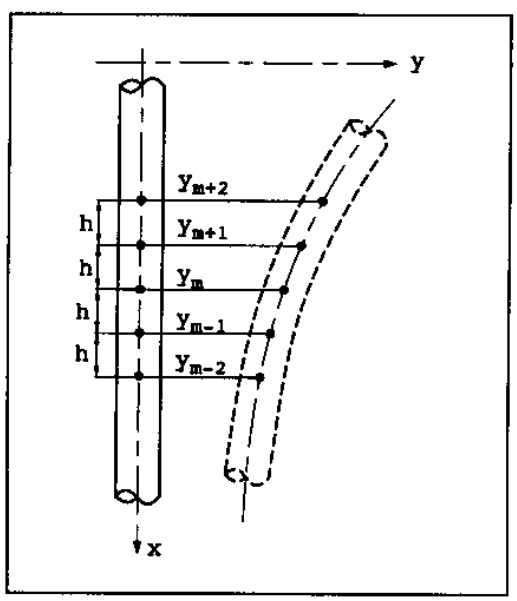

Fig. 3 Representation of deflected pile

The solution proceeds as illustrated in Fig. 4. Figure 4a shows a pile subjected to a lateral load. Figure 4b shows a family of *p-y* curves where the curves are in the 2nd and

4th quadrants because soil resistance is opposite in direction to pile deflection. Also in Fig. 4b is a dashed line showing the deflection of the pile, either assumed or computed on the basis of an estimated soil response. Figure 4c shows the upper *p-y* curve enlarged with the pile deflection at that depth represented by the vertical, dashed line. A line is drawn to the soil resistance *p* corresponding to the deflection *y* with the slope of the line indicated by the symbol E_S. Figure 4d shows the values of E_S plotted as a function of *x*. In performing a computation, the computer utilizes the computed values of E_S and iterates until the differences in the deflections for the last two computations are less than a specified tolerance. If desired, bending moment along the pile can be computed during iterations, using the appropriate difference equation, and the value of *EI* can be computed and varied along the pile with each iteration.

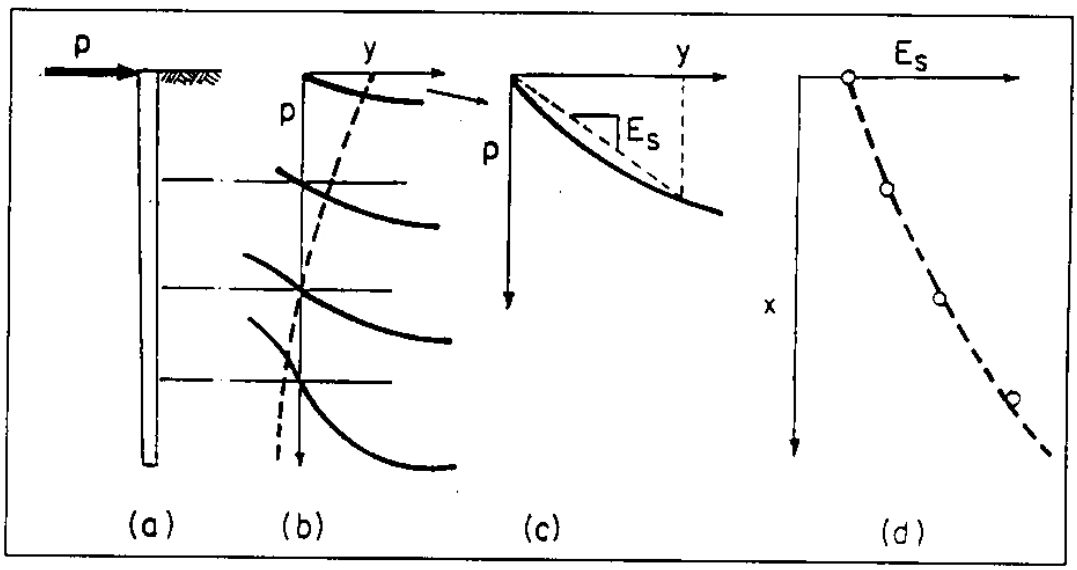

Fig. 4 Procedure for solving for response of a laterally loaded pile

After deflections have been computed, difference equations can be employed to compute rotation, bending moment, shear, and soil reaction as a function of *x*. The number of iterations for a tolerance of 0.00025 mm is usually less than 20. A high-speed computer can converge to a solution in less than one second of central-processor time. Thus, if *p-y* curves are available, a solution to a given problem can be obtained with little difficulty.

4. Soil Response Curves

With regard to soil response, the assumption is made that there is no shear stress at the surface of the pile parallel to its axis (the direction of the soil resistance is perpendicular to the axis of the pile). Any error due to this assumption is thought to be negligible.

The three factors that have the most influence on a *p-y* curve are the soil properties, the pile diameter, and the nature of loading. The correlations that have been developed for predicting soil response are based on the best estimate of the properties of the in situ soil with no adjustment for the effects of the method of installation. The logic is that the zone of soil close to the pile wall is mainly influenced due to installation, while a mass of soil of several diameters from the pile is stressed as lateral deflection occurs. There are instances, of course, where the method of pile installation must be considered; for example, if a pile is jetted into place, a considerable volume of soil could be removed with a significant effect on the soil response.

The *p-y* curves are strongly responsive to the nature of the loading. Recommendations have been developed for predicting curves for short-term static loading and for cyclic (or repeated) loading. However, there are no current recommendations for the cases where the loading is dynamic or sustained. Recommendations where the inertia of the soil is considered are needed because of the necessity for rational methods of analyzing pile-supported structures under earthquake loadings. With regard to sustained loadings on the piles supporting a retaining wall, for example, the problem is complex if the piles are driven into soft clay. The problem must be solved as a whole to account for three-dimensional consolidation and time-dependent changes in loading.

Soil-response curves have been obtained from several full-scale experiments. The piles were instrumented for the measurement of bending moment as a function of depth. Loads were applied in increments and a bending-moment curve was obtained for each load. Two integrations of each curve yielded pile deflection and two differentiations yielded soil reaction (Matlock and Ripperger, 1958). The cross-plotting of deflection and soil resistance yielded experimental *p-y* curves.

Methods for predicting *p-y* curves have been worked out for soft clay (Matlock, 1970), for stiff clay below the water surface (Reese et al., 1975), for stiff clay above the water table (Welch and Reese, 1972), and for rock (Nyman, 1980). Several authors have made use of reports in the technical literature on instrumented tests and on uninstrumented tests to make other recommendations (Parker and Reese, 1971; Sullivan, 1977; Bhushan et al., 1981; O'Neill and Murchison, 1984; O'Neill and Gazioglu, 1984).

5. Case Study

Price and Wardle (1987) reported the results of lateral-load tests of a bored pile, identified as TP12, with a length of 12.5 m and a diameter of 1.5 m. The location of the tests was not given and is listed as the location of the Building Research Establishment for convenience. The reinforcement consisted of 36 round bars, 50 mm in diameter, on a 1.3-m-diameter circle. The yield strength of the steel was 425 N/mm^2. The cube strength of the concrete was 49.75 N/mm^2. The bending moment at which a plastic hinge would occur was computed to be 15,900 kN-m at concrete strain of 0.003.

The authors installed highly precise instruments along the length of the pile. The readings allowed the determination of bending moment with considerable accuracy.

The properties of soil reported by the authors, and the interpretations used for the following analyses, are shown in Table 1.

The lateral load was applied at 0.9 m above the ground line. Each load was held until the rate of movement was less than 0.05 mm in 30 minutes. The load was reduced to zero in stages and held at zero for one hour. Computer Program *LPILE* was used in computing the response of the pile with the conditions indicated.

Table 1. Reported properties of soil at Garston

Depth m	Description	N_{SPT}	Unit weight kN/m^3	Friction angle degrees
0-0.36	Fill	18	---	---
0.36-3.5	Dense sandy gravel	65	21.5	43
3.5-6.5	Coarse sand and gravel	30	9.7	37
6.5-9.5	Weakly cemented sandstone	61	11.7	43
9.5-	Highly weathered sandstone	140	---	---

The comparisons of pile-head deflection and maximum bending moment are shown in Fig. 5. The curves for deflection show that the computation is about 20% unconservative for the larger loads and in good agreement for the smaller loads. The maximum bending moment from the experiment is about 12% higher than the computed value at the same lateral load. The computer yielded a lateral load of 4,520 kN to cause a plastic hinge.

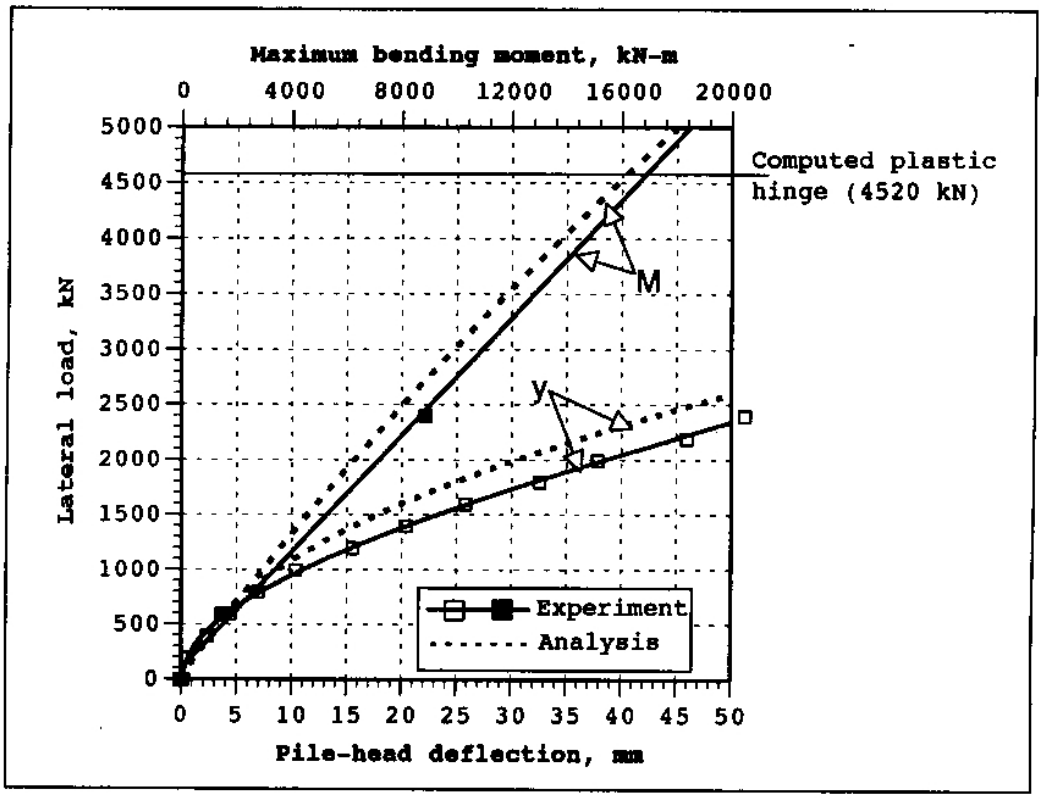

Fig. 5 Comparison of experimental and computer values of maximum bending moment and pile-head deflection, static loading, Garston

6. Summary of Several Case Studies

Reese and Van Impe (2001) presented the results for a number of case studies (pp. 259-302) and developed Fig. 6 for comparison of experimental and computed values of maximum bending moment at the service load for the tests. Excellent agreement was found for a wide range of loads.

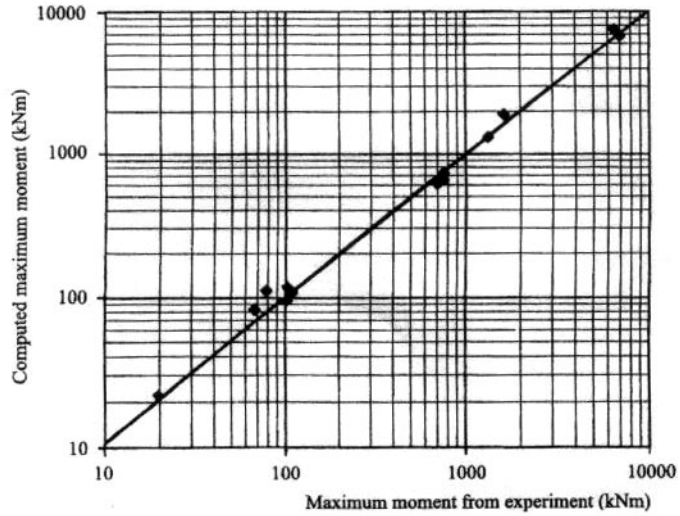

Fig. 6 Comparison of experimental and computed values of maximum bending moment at service load for various tests

Reese and Van Impe extended the case studies to compare experimental and computed values of pile head deflection at the service load. The results are shown in Fig. 7 and are not as striking as for maximum bending moment. The results were reasonable close or conservative except for the test with the largest deflection from experiment. The loading for that test was repeated.

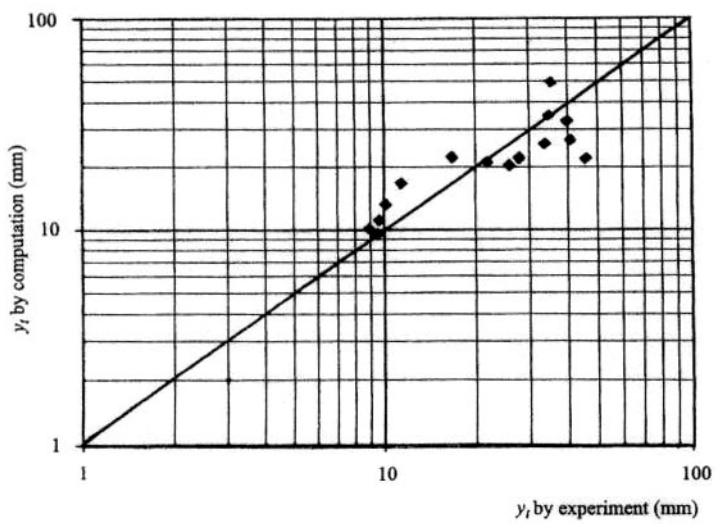

Fig. 7 Comparison of experimental and computed values of pile-head deflection at service load for various tests

7. Comment on Relevance of the Selected Model for the Soil

The model used for the soil is the so-called "Winkler" model, where the soil is modeled by discrete mechanisms. Some reviewers have suggested that the model violates the equations of continuity for the soils. To counter those arguments and to state that the model has proven to be adequate, Matlock (1970) performed tests with a pile with a free head in one case with a fixed head in another. The *p-y* curves obtained from the two experiments were essentially the same even though the pattern of lateral deflection of the pile was markedly different in the two cases.

Secondly, the *p-y* curves that were developed are based in cases where the continuum effect was fully satisfied. Thirdly, the results of a number of case studies show that the agreement between experiment and computation are well within the range of accuracy one would expert in foundation engineering.

8. Concluding Comments

A perusal of the preceding material reveals that the user of *LPILE* is presumed to be knowledgeable in the civil engineering topics of geotechnical engineering, structural engineering, and engineering mechanics. Geotechnical engineers recognize that soil is a complex and nonlinear material, that the character in the soil is influenced by many variables including its detailed history, and that its response is influenced by the nature of loading and changes in the environment.

Geotechnical engineers further recognize that a variety of methods may be used to obtain the numerical characteristics of the *in-situ* soil. The methods include a variety of laboratory tests on samples that may have been altered during extraction and a variety of *in-situ* tests. Numerical values of these several tests seldom agree closely. The geotechnical engineer will take into account many factors is selecting values of the *in-situ* soil for use in design.

A further consideration of great importance to the knowledgeable geotechnical engineer is that the installation of deep foundations will influence greatly the properties of the soil surrounding the deep foundation and that these properties may be changing with time. For example, a pile driven into saturated clay will cause the development of

excess pore-water pressures that will dissipate with time. The time-dependent dissipation results in changes in the properties of the clay that are taken into account in design.

An additional consideration that is addressed by the knowledgeable geotechnical engineer is that no recommendations for the response of piles under lateral load are available for some soils. For example, definitive tests of piles in calcareous soils that are internally cemented in some horizons are generally unavailable and *p-y* curves must be estimated, based on the available information. The geotechnical engineer may on occasion make a strong recommendation for the performance of full-scale load tests at the site in question, preferably with the piles instrumented for the measurement of deflection and soil resistance at close intervals along the length of the test pile.

Similar requirements, though perhaps lest strenuous, exist for the engineers with specialties in structural mechanics and engineering mechanics. In the first instance, the engineer will assure that the nonlinear properties of the structural members have been modeled properly, and in the second instance will assure that correct solutions have been obtained to the complex and nonlinear differential equations.

Even though some complexities are indicated in the analysis of deep foundations under lateral loads, a number of features exist that favor the user. First, solutions of the nonlinear beam-column equation are made very rapidly by the modern personal computer. The rapid solutions allow the user to investigate the importance of variables in the representation of the soil and in the representation of the bending stiffness of the deep foundation. Such trials can assist immensely in the selection of appropriate parameters.

Second, the same code or similar codes are in use by engineers in every State of the United States and in over forty other countries. Thus, many users exist who are able to share information on the successful use of the codes.

Third, Ensoft supports its software and maintenance updates are frequently issued with the view of making the software "more friendly." Many questions are answered and consulting services are available for the solution of complex problems.

Fourth, a large amount to technical literature on the subject of deep foundations under lateral loading is available. The user is urged to make use of these resources.

In the end, many "helps" are available to the user in the successful use of the code for the solution of the complex problem of a pile or drilled shaft under lateral load, but the proper use of the computer code for the solution of the problem is the responsibility of the user.

9. Concluding Comments

Several steps have been undertaken in-house by Ensoft, Inc. to verify the output of program *LPILE*. The user, if desired, may easily perform some of the elementary computations shown below.

1. With regard to the static equilibrium of the lateral forces on a single pile, the values of soil resistance can be computed and plotted along the length of the pile. With the lateral loads at the top of the pile, a check on the equilibrium of lateral forces can be made. A satisfactory check has been made by estimation; a more comprehensive check can be made by use of numerical integration of the distributed loads. The program conducts such checks internally to ensure force equilibrium.
2. The final internal check relates to the computed movement of the system. The first step is to refer to the computer output to confirm that the distributed load (soil resistance) and the distributed deflections along the length of the pile are consistent with the *p-y* curves that were input. If equations were used to compute the values of *p* and *y*, it is necessary to interpret the equations at a sufficient number of points to shown that the soil criteria for lateral load was followed. The second step with respect to lateral load is to employ the diagram in Step 1 and to use principles of mechanics to ascertain that the deflection of the pile was computed correctly.

While employing the steps shown above have confirmed the internal functioning of program *LPILE*, the application of the program to results of field experiments is

useful. As noted earlier, the book by Reese & Van Impe (2001)[1] presents a discussion of the development of the methods used in program *LPILE* and applies the methods to several cases.

Although the program has been used with apparent success in many analyses. New information is being developed and new versions may be written from time to time. No warranty, expressed or implied, is offered as to the accuracy of results from the program. The program should not be used for design unless caution is exercised in interpreting the results and independent calculations are available to verify the general correctness of the results. All users are requested to inform Ensoft, Inc. immediately of any errors that are believed to exist in the coding so it can be studied and corrected if necessary.

Ensoft, Inc. usually verifies the solution produced by program *LPILE* with hand-calculation examples and comparisons with test data from selected instrumented load tests.

[1] Reese, L. C., & W. F. Van Impe, *Single Piles and Pile Groups Under Lateral Loading*, Balkema, Rotterdam, 2001, 463 pages.

REFERENCES

American Petroleum Institute, "Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms, API Recommended Practice 2A (RP 2A)," Seventeenth Edition, April 1, 1993.

Baguelin, F., J. F. Jezequel, and D. H. Shields, *The Pressuremeter and Foundation Engineering*, Trans Tech Publications, 1978.

Bhushan, K., L. J. Lee, and D. B. Grime, "Lateral Load Test on Drilled Piers in Sand," Preprint, ASCE Annual Meeting, St. Louis, Missouri, October 26-30, 1981.

Broms, B. B., "Design of Laterally Loaded Piles," *Proceedings*, American Society of Civil Engineers, Vol. 91, No. SM3, May, 1965, pp. 77-99.

Det Norske Veritas and Riso National Laboratory, *Guidelines for Design of Wind Tunnels,* 2001,253 pages.

Det Norske Veritas, "Rules for the Design, Construction, and Inspection of Offshore Structures," Veritsveien 1, 1322 Hovek, Norway, 1977.

George, P. and D. Wood, *Offshore Soil Mechanics*, Cambridge University Engineering Department, 1977.

Georgiadis, M., "Development of p-y Curves for Layered Soils," *Proceedings,* Geotechnical Practice in Offshore Engineering, American Society of Civil Engineers, April, 1983, pp. 536-545.

Jamiolkowski, M., "Design of Laterally Loaded Piles," General Lecture, International Conference on Soil Mechanics and Foundation Engineering, Tokyo, Japan, 1977.

Matlock, H., "Correlations for Design of Laterally-Loaded Piles in Soft Clay," Paper No. OTC 1204, *Proceedings*, Second Annual Offshore Technology Conference, Houston, Texas, Vol. 1, 1970, pp. 577-594.

Matlock, H. and A. T. Haliburton, "Finite-Element Method of Solution for Linearly Elastic Beam Columns," Research Report No. 56-1, Center for Highway Research, The University of Texas at Austin, Austin, Texas, September, 1966.

Matlock, H. and W. B. Ingram, "Bending and Buckling of Soil Supported Structural Elements," Paper No. 32, *Proceedings*, Second Pan American Conference on Soil Mechanics and Foundation Engineering, Brazil, July, 1963.

Matlock, H. and L. C. Reese, "Generalized Solution for Laterally Loaded Piles," *Transactions,* American Society of Civil Engineers, Vol. 127, Part I, 1962, pp. 1220-1251.

Matlock, H. and E. A. Ripperger, “Measurement of Soil Pressure on a laterally Loaded Pile,” Proceedings, American Society for Testing Materials, Vol. 58, 1958, pp. 1245-1259.

Nyman, K. J., “Field Load Tests of Instrumented Drilled Shafts in Coral Limestone”, Unpublished Masters Thesis, The University of Texas at Austin, May 1980, 181 pages.

O’Neill, M. W. and S. M. Gazioglu, “Evaluation of p-y Relationships in Cohesive Soil,” *Proceedings*, Symposium on Analysis and Design of Pile Foundations, ASCE, San Francisco, October 1-5, 1984, pp. 192-213.

O’Neill, M. W. and J. M. Murchison, “Evaluation of p-y Relationships in Cohesionless Soils,” *Proceedings*, Symposium on Analysis and Design of Pile Foundations, ASCE, San Francisco, October 1-5, 1984, pp. 174-191.

Parker, F., Jr. and L. C. Reese, "Lateral Pile-Soil Interaction Curves for Sand," *Proceedings,* The International Symposium on the Engineering Properties of Sea-Floor Soils and Their Geophysical Identification, The University of Washington, Seattle, Washington, July, 1971.

Poulos, H. G. and E. H. Davis*, Pile Foundation Analysis and Design*, Wiley, New York, 1980.

Price, G. and I. F. Wardle, "Lateral Load Tests on Large Diameter Bored Piles," Contractor Report 46, Transport and Road Research Laboratory, Department of Transport, Crowthorne, Berkshire, England, 1987, 45 pages.

Reese, L. C., "Analysis of a Bridge Foundation Supported by Batter Piles," *Proceedings*, Fourth Annual Engineering and Geology and Soils Engineering Symposium, Moscow, Idaho, April, 1966, pp. 61.

Reese, L. C., "The Analysis of Piles under Lateral Loading," *Proceedings,* Symposium on the Interaction of Structure and Foundation, The Midland Soil Mechanics and Foundation Engineering Society, University of Birmingham, Birmingham, England, July, 1971, pp. 206-218.

Reese, L. C., “Handbook on Design of Piles and Drilled Shafts under Lateral Load,” a report prepared for Federal Highway Administration, U.S. Department of Transportation, Research, Development and Technology, McLean, VA, FHWA-IP-84-11, July, 1984, 360 pages.

Reese, L. C., W. R. Cox, and F. D. Koop, "Analysis of Laterally Loaded Piles in Sand," Paper No. OTC 2080, *Proceedings,* Fifth Annual Offshore Technology Conference, Houston, Texas, 1974 (GESA Report No. D-75-9).

Reese, L. C., W. R. Cox, and F. D. Koop, "Field Testing and Analysis of Laterally Loaded Piles in Stiff Clay," Paper No. OTC 2313, *Proceedings,* Seventh Offshore Technology Conference, Houston, Texas, 1975.

Reese, L. C. and H. Matlock, "Numerical Analysis of Laterally Loaded Piles," *Proceedings,* Second Structural Division Conference on Electronic Computation, American Society of Civil Engineers, Pittsburgh, Pa., 1960, pp. 657.

Reese, L. C., and W. F. Van Impe, *Single Piles and Pile Groups Under Lateral Loading,* Balkema, 2001, 463 pages.

Sullivan, W. R., "Development and Evaluation of a Unified Method for the Analysis of Laterally Loaded Piles in Clay," Unpublished Masters Thesis, The University of Texas at Austin, May, 1977.

Welch, R. C. and L. C. Reese, "Laterally Loaded Behavior of Drilled Shafts," Research Report No. 3-5-65-89, conducted for Texas Highway Department and U.S. Department of Transportation, Federal Highway Administration, Bureau of Public Roads, by Center for Highway Research, The University of Texas at Austin, May, 1972.